

Mathematics Olympiad Pennsylvania State University Hazleton

SPRING 2012

Round One

Problem 1. How many numbers among $1, 2, \dots, 2012$ are not multiples of any of 3, 5, or 7?

Problem 2. Assume that you have two fuses each of which burns out in one hour, not necessarily at constant speeds. Is it possible to use these fuses to time 45 minutes?

Problem 3. Find the minimum number N such that among any N positive integers there will always be at least one pair with the sum or difference divisible by 2012.

Problem 4. Find such a rearrangement of the set $\{1, 2, \dots, 100\}$ that any subsequence of length 11 of that sequence is neither increasing nor decreasing.

Problem 5. Show that every polynomial is the difference of two increasing polynomials.

Problem 6. Show that for every positive integer n the number $5^{5n+1} + 4^{5n+2} + 3^{5n}$ is a multiple of 11.