Mathematics Olympiad

PSU Hazleton, Spring 2008

(First round due March 18)

1. Show that among any 2008 integers it is always possible to find a group of numbers with the sum divisible by 2008.

2. Let $x + \frac{1}{x}$ be an integer. Show that $x^n + \frac{1}{x^n}$ is also an integer for any integer n.

3. Show that the number $111\ldots111$, consisting of 81 digits all of them equal to 1, is divisible by 81.

4. A group of six females contains exactly 11 pairs of acquintances (that is people that know each other). You share a hot stock tip with one of these females. Assuming that every female shares a tip with all of her friends, show that everybody in the group will find out about this excellent buying opportunity.

5. Each side of a triangle is divided in three equal parts and then every vertex of this triangle is connected with the division point on the opposite side which comes first in the counter clock order. In this way, a smaller triangle is formed inside the original triangle. Find the area of a smaller triangle as a fraction of the area of the original triangle.