Mathematics Olympiad
Pennsylvania State University Hazleton

SPRING 2012

Round Two

Problem 1. Let m and n be positive integers. Exhibit an arrangement of the integers from 1 to mn which has no increasing subsequence of length $m+1$, and no decreasing subsequence of length $n+1$.

Problem 2. The sum of ten integers is zero. Show that the sum of the fifth powers of these numbers is divisible by 5.

Problem 3. There are 2012 refueling stations along a circular path such that the total amount of fuel on all of them is enough for a certain vehicle to cover the whole distance along this path. Assuming that, at each station, the vehicle can consume all the fuel available there, show that it is possible for this vehicle to start at some station and cover the whole distance along this path.

Problem 4. An equilateral triangle was cut into a number of equilateral triangles in such a way that the perimeters of all smaller triangles were integers. Prove that the perimeter of the original triangle was also integer.