Mathematics Olympiad
Pennsylvania State University Hazleton

SPRING 2010

Round One

Problem 1. How many girls were there among the 42 participants at a dancing party if the first girl danced with 7 boys, the second girl danced with 8 boys, and so on so that the last girl danced with all the boys?

Problem 2. There are three groups of matches on the table: a group of 2009, a group of 2010, and a group of 2011 matches. A game of two players is such that they both take turns and in one turn a player is allowed to remove one match from each of the two groups of matches chosen by the player. The player that takes the last match from one of the groups of matches wins the game and game ends. Which player can win this game? Describe a strategy for wining.

Problem 3. The game "Battleships" is played on a square board consisting of 49 fields. A ship in the shape of a rectangle of size 2×3 is placed on the board. Find the minimum number of hits that would suffice to hit this ship.

Problem 4. Find all pairs of natural numbers x and y such that $\dfrac{xy^2}{x+y}$ is a prime number.

Problem 5. Find the minimum value of the function

$$f(x)=|x-1|+|x-2|+\ldots+|x-2010|.$$

Problem 6. Show that $2^{3^{2009}}-1$ is divisible by 3^{2010} and is not divisible by 3^{2011}.